

Dynamic Weighting of Laser Tracker Measurements for Bundle Adjustment

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Abstract: A laser tracking interferometer system provides real-time three-dimensional measurements in large volumes. It is portable and easily moved to the measurement site. Laser trackers are spherical measurement systems that measure a three-dimensional location of a retro-reflector. The bundle adjustment is a method that computes coordinates from laser tracker measurements. It maximizes the accuracy by allowing each individual measurement to be weighted based upon their type and accuracy.

This study will define the bundle adjustment angle and range measurement uncertainty dependence relative to the distance between the Laser Tracker and the retro-reflector. The study shows that by weighting the range and angle data with appropriate functions, the residuals of the measurements are smaller and thus the uncertainty of point coordinate data is improved. The study presents test data collected in control laboratory environment, then confirms the improvements by comparing bundle adjustment results of real data with and without the modified weighting approach.

1. INTRODUCTION

Bundle adjustment is a computational method that covers a wide range of measurement configurations. It allows for introducing different types of measurements and constraints. Additionally the type of instrument can vary along with the number of instrument locations used in measurement volumes that may vary significantly. For the measurement of large volumes the bundle adjustment is the best tool to define a consistent world coordinate system when multiple instrument positions are used to create the survey.

This paper focuses on laser tracking interferometer measurements, i.e. horizontal angles, vertical angles and ranges as input to a bundle adjustment. In these surveys it is typical for the range between target and instrument to vary greatly. In order to obtain the best possible results for this situation an investigation has been carried out to demonstrate the potential improvements of dynamic weighting (i.e., range dependent weighting) over static weighting (i.e., independent of range) of laser tracker measurements for bundle adjustments.

2. BUNDLE ADJUSTMENT

A bundle adjustment is a numerical algorithm used to refine redundant measurements from triangulation, spherical measurement systems or other dimensional measurement system into the best possible point coordinates. These measurement systems include photogrammetry, videogrammetry, theodolites, laser trackers, etc. The technique assumes redundant measurements are available, then it solves for best possible instrument and point positions in which the sum of the squares of the measurement residuals is minimized.

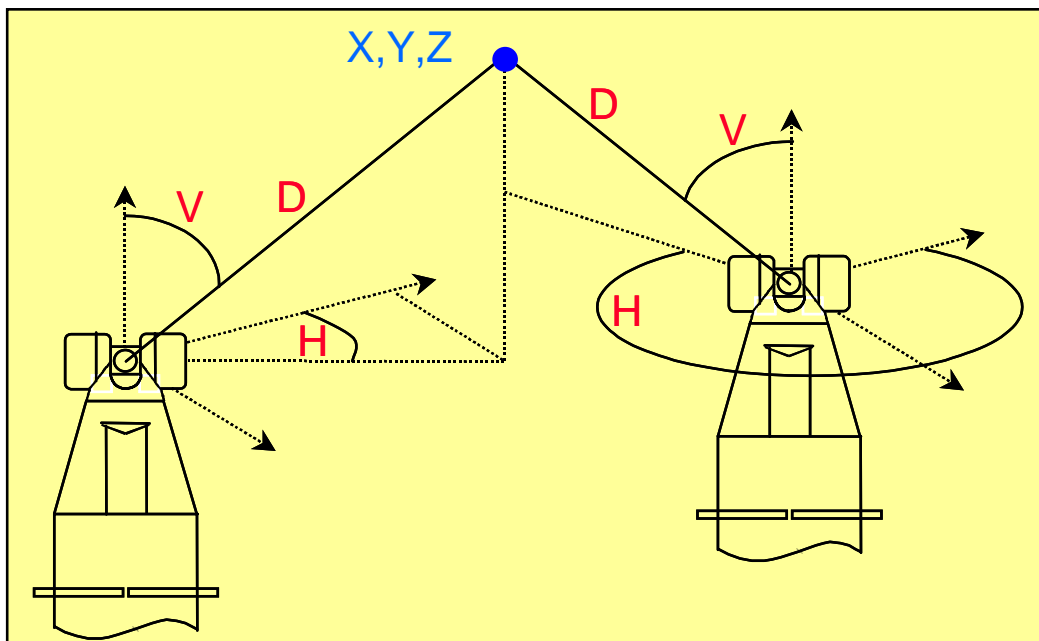


Figure 1 : Bundle adjustment; coordinates X, Y, Z of blue point are computed from measurements H, V, D of 2 laser tracker stations

In that laser tracker systems are line-of-sight systems, measurements of large structures with laser trackers are typically done with multiple station setups (e.g., one station on each side of a large object). The measurements from each position around the object are combined together to form a complete model of the object. The measurements from the different station positions can be collected by either operating several trackers at the same time or by moving one tracker around to each position. The measurements from 2 stations generate redundancy, because each laser tracker measurement (a range and two angles) yields 3D information for the point, therefore for the 3 unknowns, 6 measurements are available yielding a general uncertainty improvement by a factor of the square root of two. The bundle adjustment is the numerical mechanism that is able to effectively utilize the redundancy, and allows for the computation of residuals, which in turn can be used to verify the assumed accuracy of the measurements.

The relative accuracy of measurements used in the Bundle Adjustment is accounted for by weighting the more precise measurements higher than the less precise measurements. Traditionally laser tracker measurements were weighted statically, meaning each angle measurement was weighted the same. Similarly, each range measurement was weighted with the same value. It can be shown that the range and angle measurement accuracy are range dependent, and as such, a refinement to the weighting scheme could yield improved results. This refinement is called dynamic weighting. Dynamic weighting applies weights that are consistent with the expected range dependent inverse of the square of the measurements standard error. The new scheme for weighting laser tracker data for bundle adjustment is investigated and the dependency is characterized in this paper.

2.1. INPUT OF UNCERTAINTY TO BUNDLE ADJUSTMENT

The traditional form of input measurements and their respective standard deviations is:

$$\begin{array}{ll} H \pm \sigma_H & \text{(Horizontal Angle)} \\ V \pm \sigma_V & \text{(Vertical Angle)} \\ R \pm \sigma_R & \text{(Range)} \end{array}$$

The bundle adjustment propagates standard deviations of the coordinates and computes residuals from these quantities. In a perfect world, the standard deviations of the measurements are normally distributed and free of any systematic errors. In other words, the instrument produces perfect spherical coordinates with the exception of repeatability errors. There is, of course, no instrument in the world, which is perfect like this. However, to get as close as possible to this perfection, tracker alignment is carried out.

If the accuracy of a single pointing becomes range dependent, the standard deviations are modified as shown:

$$\begin{array}{l} H \pm \sigma_H(R) \\ V \pm \sigma_V(R) \\ R \pm \sigma_R(R) \end{array}$$

2.2. STATIC WEIGHTING VS. DYNAMIC WEIGHTING

Dynamic weighting is expected to produce “better results” because the influence of less accurate measurements is reduced and the influence of more accurate measurements is increased. “Better results” in the context of bundle adjustment means

- more realistic point coordinates and station parameters including standard deviations due to more realistic standard deviations of the measurements;
- smaller residuals and derivatives of the residuals.

3. NUMERICAL INVESTIGATION OF RANGE DEPENDENCY

Before the concept of dynamic weighting is implemented in the bundle adjustment, appropriate parameters for weighting functions must be investigated. For this purpose, a series of test measurements were performed to establish the range dependency of angle and range accuracy.

Paragraphs 3.1 “Range Accuracy Depending on Range” and 3.2 “Angle Accuracy Depending on Range” contain the following items with specific respect to angles and ranges:

- explanations on reasons why laser tracker measurements are range dependent;
- assumptions which apply to the investigations;
- a description of the collected data;
- conclusions derived from the data.

3.1. RANGE ACCURACY DEPENDING ON RANGE

Most laser trackers are equipped with 2 different units for range measurements. This paper deals only with the laser interferometer unit.

The accuracy of a laser interferometer decreases with increasing range. The accuracy degradation is primarily due to environmental conditions. The stabilization of the wavelength by electronics and firmware is another influencing factor [1],[2].

Several additional influences sometimes but not always apply; and if they do so, they overlap with the pure instrument accuracy. These influences include

- stability of the floor;
- the reflector properties; refer also to [3], [4];
- re-positioning of the reflector between the repeated measurements;
- stability of the reflector holding;

Moreover, it is impossible to accurately state to which extent these influences are systematic. Therefore instrument manufacturers tend to report conservative accuracies. For the Leica LT500/LTD500 a value of 2.5 ppm is reported. In general a sampling strategy where the range measurement is averaged over a period of time (e.g., 2-seconds) used to minimize the random variation induced by the environment.

Repeated range measurements of several distances have been carried out to verify the accuracy of range measurements with the laser tracker.

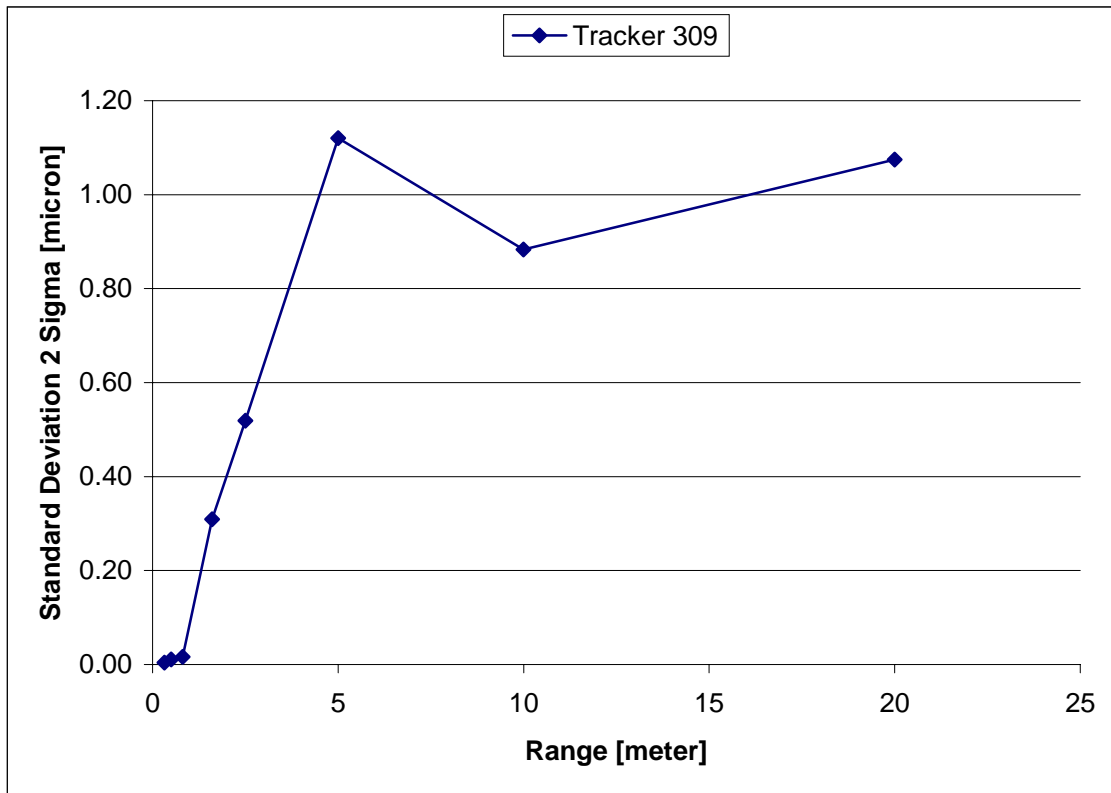


Figure 2 : Range Accuracy with Respect to Range

Figure 2 is typical of repeated measurements taken at 8 different ranges (taken with 80 samples at 80 millisecond intervals each). It averages two face measurements at horizontal directions of 0 and 180 degree. The number of measurements per range per face has been 10. The increasing standard deviation of the range accuracy is shown. The ppm value derived from this test measurement is much smaller than 2.5 because environmental disturbances and reflector related influences have been kept very small in a laboratory kind of environment.

From this and other data a function is derived which can be used to compute the standard deviation as introduced to the bundle adjustment:

$$\sigma_R = d * R$$

with d being the ppm value. Refer to the square and diamond function in Figure 9. This function has been implemented in the Leica Axyz bundle adjustment; the parameter d is editable for working out the best value for an actual job. Refer to paragraph 4 “Numerical Verification of Dynamic Weighting within Bundle Adjustment” for the results obtained from the bundle adjustment with this implementation. A value, which is derived from the test data, is:

$$d = 1 \text{ ppm}$$

3.2. ANGLE ACCURACY DEPENDING ON RANGE

While the accuracy of range measurements carried out with electronic range measurement devices is well investigated and proven for decades, the range dependency of the accuracy of angles has not been looked at that often. Traditionally, the angular accuracy is presumed to be static. The reason for this is that varying accuracy of angles applies to very close instrument range only, as shown later in this paper.

There are reasons to expect that there is some range dependency of the accuracy of angles. Uncompensated or mis-compensated beam and rotational as well as lateral axis offset errors produce increasing angle uncertainty as the range to the target decreases. As an example, given an axial offset error of 0.002 mm the angular error at 10 meters is 0.0001 decimal degrees. The same axial offset error at a range of 0.1 meters produces an angular error of 0.0011 decimal degrees. The error increase of about one order of magnitude is expected and therefore can be accounted for by weighting the observations. Refer also to [3]. Figure 3 illustrates how the angular uncertainty generated by a lateral offset is the larger the closer the reflector is to the instrument.

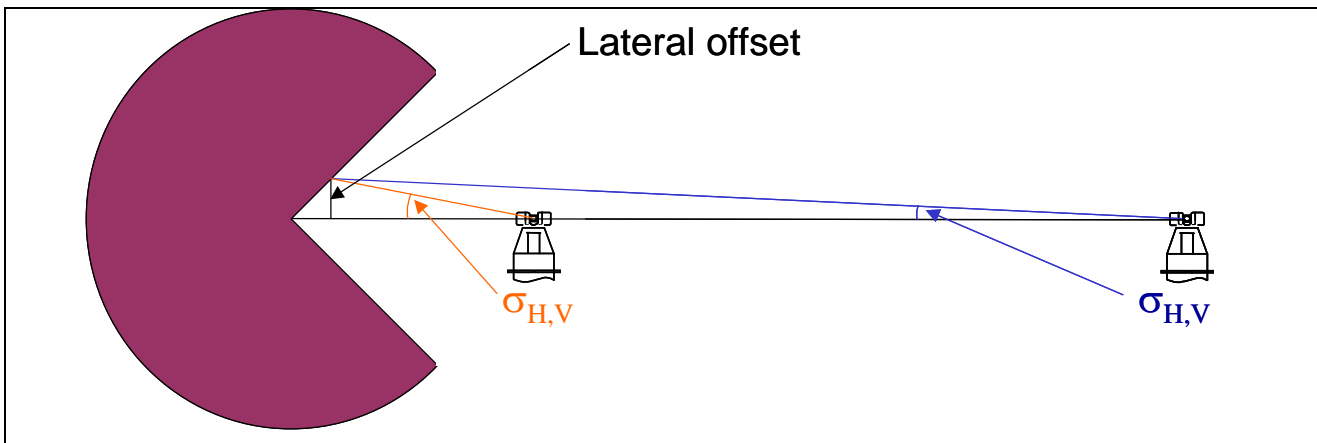


Figure 3 : Angle uncertainty originating from offset error of reflector

Like for the range accuracy, it is also true that the entire angle uncertainty is not only made up by the range dependent but also by other influences, which are the same as listed in paragraph 3.1 “Range Accuracy Depending on Range”.

To study the range dependency of angle measurement accuracy by means of test measurements various scenarios are possible; each includes bundling the data and comparing the statistics for repeated measurements at varying ranges, see Figure 4.

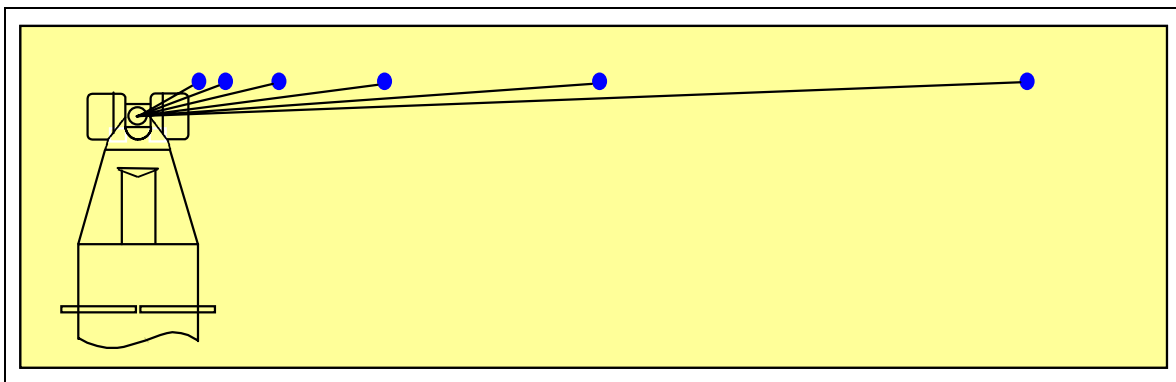


Figure 4 : Logarithmic increase of range for sample measurements

Optionally, the measurements can be carried out with two faces, at varying horizontal directions and/or with varying vertical angles. A tracker alignment should precede the measurements to remove systematic errors as much as possible.

Two test scenarios have been setup:

1. Two face measurements with 3 laser trackers at varying ranges, at 8 varying horizontal directions and, at the horizontal direction zero degrees with 5 different vertical angles;
2. One face and two face measurements with 1 laser tracker at varying ranges at 2 horizontal directions and with a vertical angle of 90 degrees.

The two face measurements have been sorted by laser tracker and range. Then the average of the two face differences as well as their standard deviations were computed. Figure 5 shows the result:

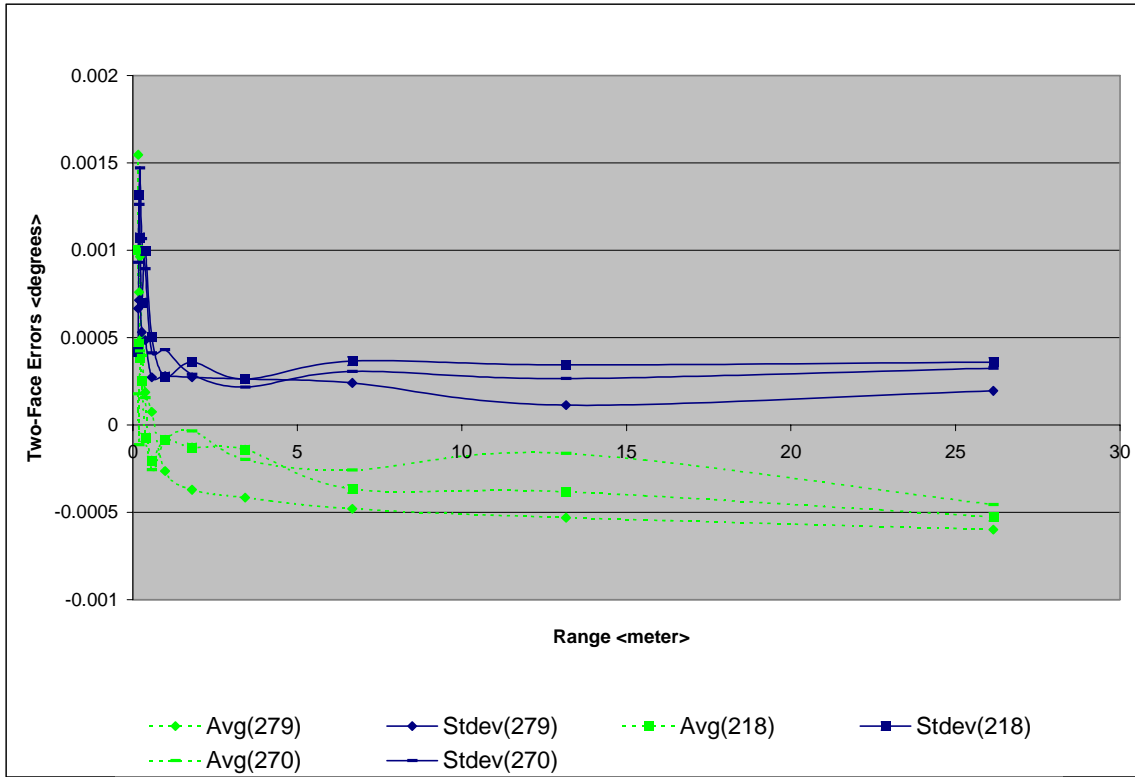


Figure 5 : Averaged two face measurements and standard deviations of angle measurements vs. range

Beside the fact that the average (dashed curves) of the two face differences contains some remaining systematic effect, the chart indicates that for close ranges up to about 2.5 meter the angle accuracy is different from the angle accuracy beyond that range. It turned out that this behavior is the same for horizontal and vertical angles.

A function has been derived from the study to compute a standard deviation for an angle measurement with respect to the range:

$$\sigma_H = a \cdot R^b$$

$$\sigma_V = \sigma_H$$

with a and b being numbers without units. R is the range in meters. Refer to the diamond function in Figure 10. This function has been implemented in the bundle adjustment; the parameters a and b are editable for working out the best values for an actual job. A pair of values, which is suggested by the test data is:

$$a = 0.001134$$

$$b = -0.404$$

Test measurements of scenario 2. focus on the repeatability of angle measurements in order to estimate the accuracy with respect to range. At each range 10 samples were taken in face 1, then 10 samples in face 2. The standard deviation for each of the two faces was computed, the same was done at a different horizontal direction.

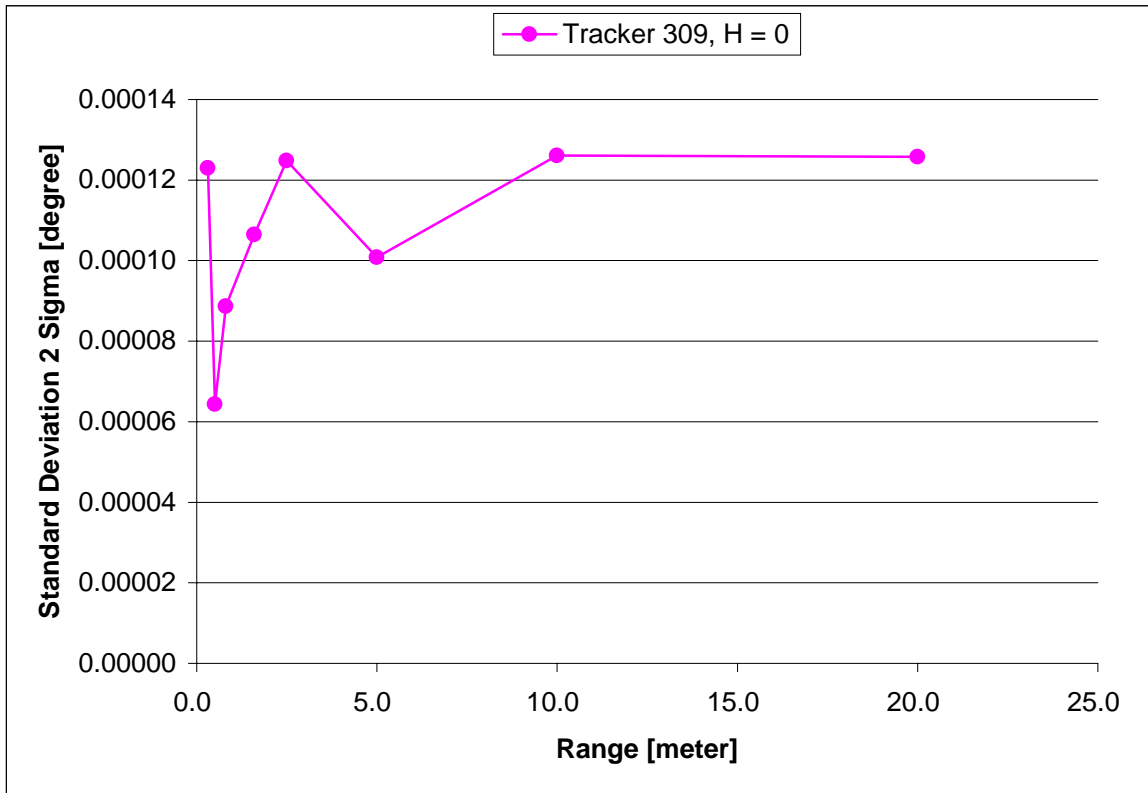


Figure 6 : Horizontal Accuracy with Respect to Range. The standard deviations of angles were obtained from repeated measurements, vs. range

A function has been derived from the data to compute a standard deviation for an angle measurement with respect to the range:

$$\sigma_H = \frac{l}{R} \cdot \frac{200}{\pi} \text{ if } R < t;$$

$$\sigma_H = f \text{ if } R \geq t$$

$$\sigma_V = \sigma_H$$

with l being a lateral offset standard deviation, t a threshold of the range for dividing the range into two areas for different computations and f being an angular standard deviation which is supposed to be constant if the range R is greater than t . Refer to the square function in Figure 10. This function has been implemented in the Leica Axyz bundle adjustment; the parameters t , l and f are editable for working out the best values for a real job.

The measurements of this scenario have been made under very good environmental conditions with very stable reflector positions. That is why the achieved level of accuracy - below 0.0001 degree - is so high.

Previous investigations under conditions, which are similar to industrial factory environments, worked out the same characteristics but on a lower level of accuracy. The three values, which are suggested by these investigations, are:

$$t = 2.5 \text{ meter}$$

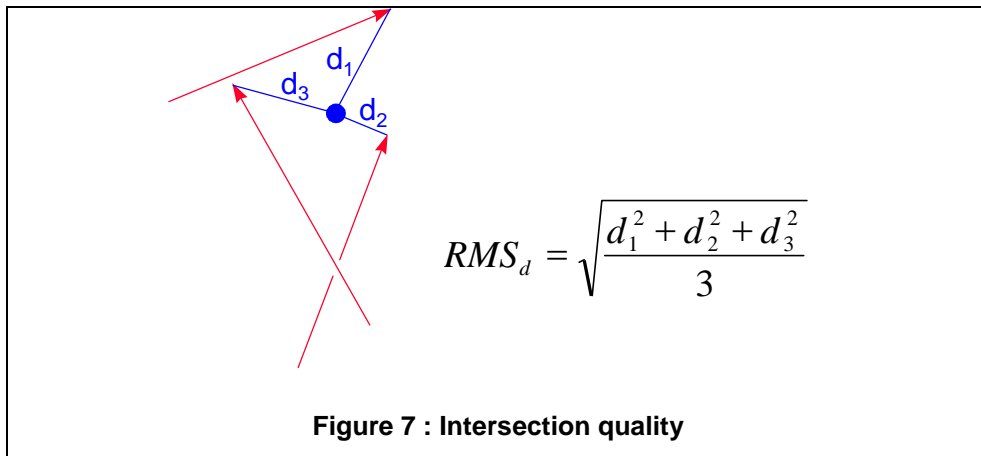
$$l = 25 \mu\text{m} = 0.001 \text{ inches}$$

$$f = 0.0005 \text{ degree}$$

4. NUMERICAL VERIFICATION OF DYNAMIC WEIGHTING WITHIN BUNDLE ADJUSTMENT

Whether the refinement of the weighting model improves the bundle adjustment can be verified by setting up a multiple tracker configuration, doing the measurements, orienting it and comparing the intersection quality

of the pointings between the various versions. Figure 7 illustrates what intersection quality means: Three pointings (with a circle) from three laser tracker stations are intersected. The bundle program adjusts the pointings to three coordinates X,Y,Z of the point (drawn as a square point). The average length of the distances d_1 , d_2 , d_3 between the 3 contributing points and the adjusted point is the RMS_D . The smaller this value the better the intersection quality. The RMS_D is sensitive to systematic and normally distributed errors; in other words, it is quite an independent check for accuracy, which has been assumed for the measurements. It is used as the criteria to consider a weighting function to have generated improvement.



4.1. CONFIGURATION OF THE BUNDLE BLOCK

A test configuration with 4 stations and major variation in range with particular attention to close ranges has been setup and measured. The measured ranges vary between 1 and 10 meter. All measurements were taken in two faces. See Figure 8.

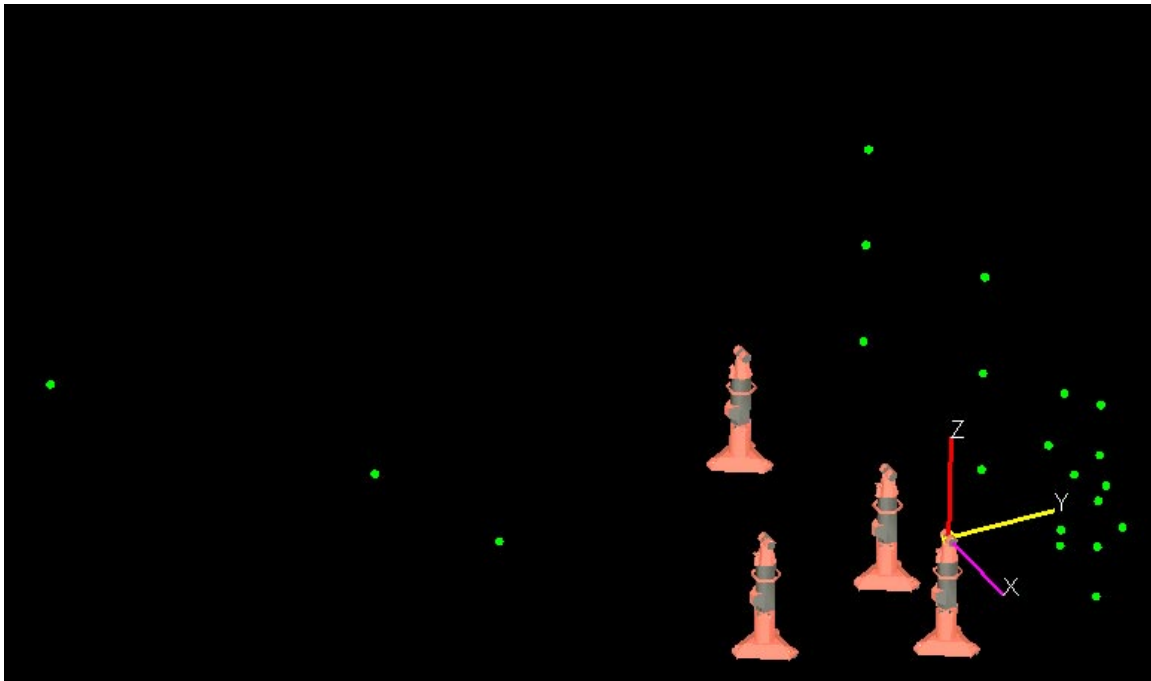


Figure 8 Test job with 4 stations

4.2. USED WEIGHTING FUNCTIONS

The block was oriented in 7 different versions. These versions cover traditional static weighting, dynamic weighting of range only, dynamic weighting of angles only and dynamic weighting of both. Table 1 shows details of the versions. Figure 9 and Figure 10 illustrate graphically the various weighting functions:

- the triangle ones are the traditional static functions;

- the square ones are the dynamic functions as suggested by the manufacturer's hardware guide [2]; the square angle accuracy function is also suggested by scenario 2. as described in paragraph 3.2 "Angle Accuracy Depending on Range";
- the diamond ones are suggested by the investigations shown in 3.1 "Range Accuracy Depending on Range" and scenario 1 in 3.2 "Angle Accuracy Depending on Range".

Version	Weighting of Range	Weighting of Angles	RMS _D [μm]
1	Static , $\sigma_R = 2.5 \mu\text{m}$	Static ; $\sigma_H = 0.0008$ degree; $\sigma_V = \sigma_H$	27
2	Dynamic , $d = 1$ ppm	Static , $\sigma_H = 0.0008$ degree; $\sigma_V = \sigma_H$	23
3	Dynamic , $d = 2.5$ ppm	Static , $\sigma_H = 0.0008$ degree; $\sigma_V = \sigma_H$	18
4	Static , $\sigma_R = 2.5 \mu\text{m}$	Dynamic , $\sigma_H = a \cdot R^b$; $\sigma_V = \sigma_H$ With $a = 0.001134$; $b = -0.404$	23
5	Static , $\sigma_R = 2.5 \mu\text{m}$	$\sigma_H = \frac{l}{R} \cdot \frac{200}{pi}$ if $R < t$; Dynamic , $\sigma_H = f$ if $R \geq t$ $\sigma_V = \sigma_H$ With $t = 2.5$ meter $l = 25 \mu\text{m}$ $f = 0.0005$ degree	23
6	Dynamic , $d = 2.5$ ppm	Dynamic , $\sigma_H = a \cdot R^b$; $\sigma_V = \sigma_H$ with $a = 0.001134$; $b = -0.404$	16
7	Dynamic , $d = 2.5$ ppm	Dynamic , like in version 5	17

Table 1

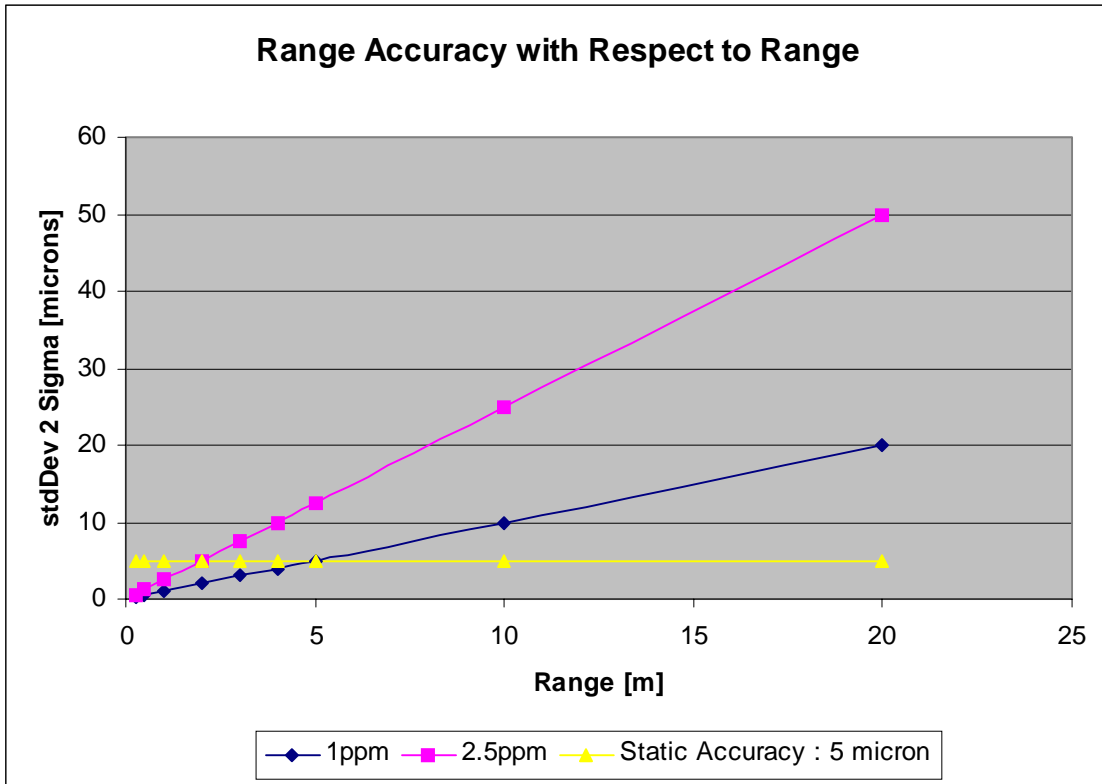


Figure 9 : Range weighting functions input to bundle adjustment

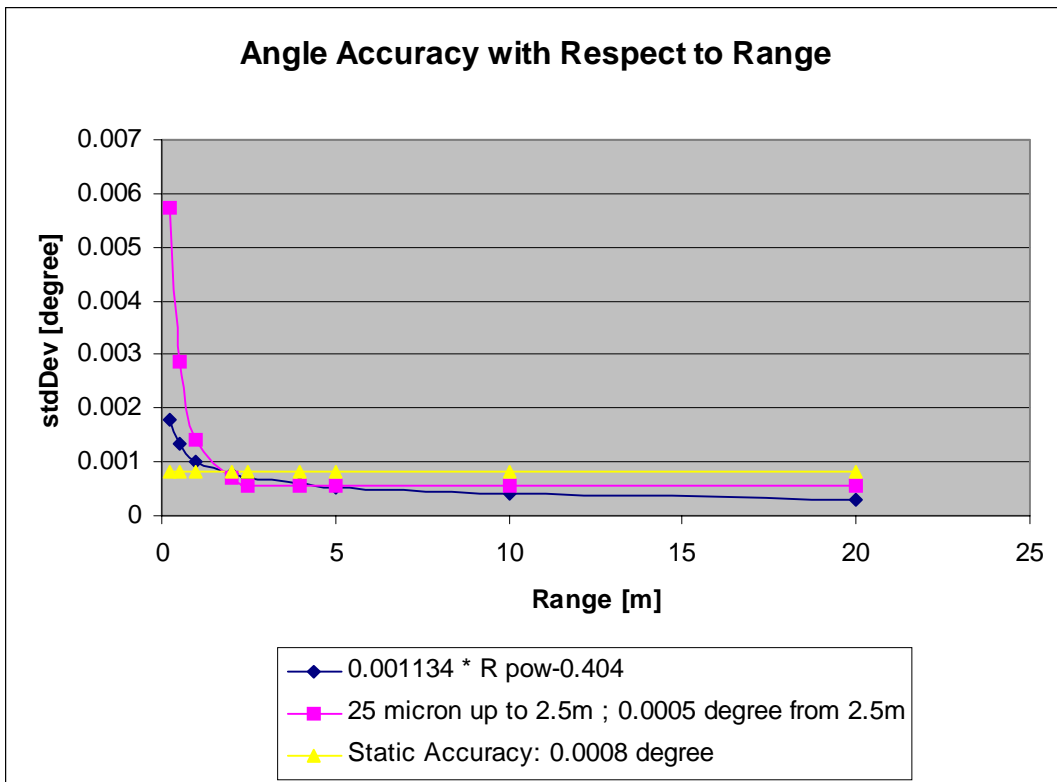


Figure 10 Angle weighting functions input to bundle adjustment

The table shows that a dramatic improvement can be achieved by dynamic weighing. The most significant contribution is obtained by the dynamic weighting of the ranges, but the improvement given by the dynamic weighting of the angles is also significant.

The documented job is based on measurements in two faces. A similar study has been made with the same job but only with measurements in one face. The achieved RMS_D values range from 21 μm to 31 μm , indicating that the level of accuracy is lower than with two face measurements but the improvements by dynamic weighting are as significant.

5. CONCLUSION

For bundle blocks with great variations in range and in particular with a significant number of measurements in very close ranges a refinement of the accuracy of the bundle adjustment can be achieved if dynamic weighting of the measurements is applied. This holds even true if a conservative level of accuracy, i.e. a level that represents a typical industrial environment is assumed.

The most significant improvement is obtained by dynamically weighting the ranges. However, the improvement achieved by dynamically weighting the angles is also remarkable.

This has been shown based on a sample bundle block. Two face measurements have been done in order to reduce the influence of any remaining systematic uncertainty. However, this is appropriate only if the reflector(s) can be re-positioned with very high accuracy.

Similar improvements by dynamic weighting can also be achieved if measurements are made in one face only.

References:

1. Greenwood, Tom; Nielsen, Tim; Sandwith, Scott: "Environmental Effects on Laser Tracker Measurements"; presented paper at the Boeing Seminar 1998; published on a CDROM which was issued as the conference proceedings.
2. Leica LT500/LTD500 hardware guide.
3. Palmateer, John: "Those #%&!\$ Corner Cubes"; presented paper at the Boeing Seminar 1998; published on a CDROM which was issued as the conference proceedings.4. Markendorf, Albert: "The Influence of the Tooling Ball Reflector on the Accuracy of Laser Tracker Measurements: Theory and Practical Tests"; presented paper at the Boeing Seminar 1998; published on a CDROM which was issued as the conference proceedings.

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